Exercise 35

(a) Find the slope of the tangent line to the curve $y = 9 - 2x^2$ at the point $(2, 1)$.

(b) Find an equation of this tangent line.

Solution

The slope of the tangent line to $y = 9 - 2x^2$ at the point $(2, 1)$ is found by calculating the derivative of $y = f(x)$ and then setting $x = 2$. Use the definition of $f'(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{[9 - 2(x+h)^2] - (9 - 2x^2)}{h} \\
&= \lim_{h \to 0} \frac{-2(x+h)^2 + 2x^2}{h} \\
&= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 2x^2}{h} \\
&= \lim_{h \to 0} \frac{(-2x^2 - 4xh - 2h^2) + 2x^2}{h} \\
&= \lim_{h \to 0} \frac{-4xh - 2h^2}{h} \\
&= \lim_{h \to 0} (-4x - 2h) \\
&= -4x
\end{aligned}$$

The desired slope is therefore

$$f'(2) = -4(2) = -8.$$

To determine the equation of the line, use the given point $(2, 1)$, this slope, and the point-slope formula.

$$\begin{aligned}
y - 1 &= -8(x - 2) \\
y - 1 &= -8x + 16 \\
y &= -8x + 17
\end{aligned}$$

Below is a graph of both $y = -8x + 17$ and $y = 9 - 2x^2$ versus x. Notice that the line is tangent to the curve at $x = 2$.